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INDIRECT SIGNALS OF SUSY IN GAUGE BOSON PAIR PRODUCTION AT LEP AND NLC *

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ABSTRACT

We compute the dominant one-loop radiative corrections to the cross-section for $e^+e^- \rightarrow V_1V_2$ ($V_{1/2} = \gamma, Z, W^{\pm}$) in the MSSM. We find that the genuine vertex corrections are very small. The oblique corrections are potentially large enough to be tested at LEP-II or NLC. However, the sensitivity is below the one from other high precision electro-weak measurements but can serve as a self-consistency check.

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1. Introduction

LEP-I experiments essentially rule out new physics below $m_z/2$ [1]. However, the high statistics allow even to constrain virtual effects from particles not kinematically accessible at LEP-I energy. Thus, it is natural to ask whether this is also possible at upcoming e^+e^- colliders. The higher center-of-mass energy, E_{cm}, opens new channels even within the framework of the standard model. In addition, the clean environment of an e^+e^- collider is ideal to test the triple gauge boson coupling (TGC). In light of the success of the Standard Model (SM) it is hard to imagine that the TGC will exhibit a deviation from the SU(2)×U(1) gauge structure. Nonetheless, it might be possible to detect anomalies indicating virtuall effects of new physics. In table 1 we have listed upcoming collider experiments. In e^+e^- colliders a pair of gauge bosons can be

Table 1. Upcoming e^+e^- collider experiments

name	Type	$\sqrt{s}$	$\int \mathcal{L}dt$	date
LEP-II	circular	$165 \sim 192$ GeV	4×0.5 fb^{-1}	now
NLC	linear	$0.5 \sim 2$ TeV	$50 \sim 200$ fb^{-1}	2005/10

produced in three possible chanels as depicted in fig. 1. Note, there is no s-chanel (no u-chanel) for $\gamma\gamma$, γZ, ZZ (W^+W^-) production at tree-level. In table 2 we have listed the cross-section

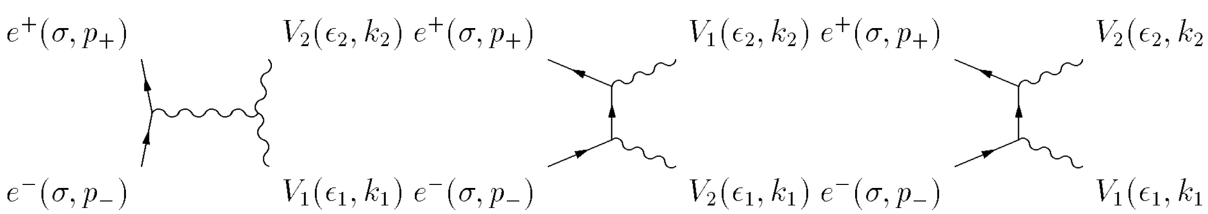


Fig. 1. Tree-level contributions to $e^+e^- \to V_1V_2$

for gauge boson pair production in an e^+e^- collider obtained by integrating the differential cross-section

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to V_1V_2) = \frac{f_S\alpha_{\mathrm{em}}^2}{4s}\sqrt{1-\frac{4m_W^2}{s}}\,|\mathcal{A}|^2\ , \tag{1}$$

where the symmetry factor $f_S = 1$ (1/2) for W^+W^- and γZ ($\gamma\gamma$ and ZZ). We also present the expected precision to which this cross-section can be measured at LEP-II ($E_{cm} = 200$ GeV) and NLC ($E_{cm} = 500$ GeV) assuming that the statistical errors dominate. With a precision in the percent range these processes are potentially sensitive to radiative corrections (RC). The agreement (disagreement) of the theoretical and experimental values provide an important self-consistency check of the SM (hint for new physics). However, before we can appreciate the importance of this process we have to answer the question whether we can actually learn anything new from testing the TGC? One of the most important achievements at LEP-II will be

Table 2. cross-sections and expected statistical errors

$\sqrt{s}$ [in GeV]	165	176	190	205	500	2000	LEP-II	NLC
WW	10.7	16.1	17.7	17.5	6.5	0.82	0.5 %	0.2%
WW_{F-B}	3.0	7.8	10.7	11.9	6.0	0.79	0.7 %	0.2%
ZZ	-	-	1.0	1.2	0.37	0.045	2.0 %	0.7%
γZ	21.1	16.6	12.7	9.9	1.1	0.064	0.6 %	0.4%
$\gamma\gamma$	5.8	5.1	4.4	3.8	0.63	0.039	1.0 %	0.6%

a very precise measurement of $m_{\rm w}$ to $\delta m_{\rm w} = \pm 50$ MeV. This will pose the strongest constraint on new physics via Δr[2] with an expected precision

$$\delta\Delta r \;=\; \pm 0.1\%(Th) \pm 1\% \frac{\delta m_{\rm w}}{170 \text{ MeV}} \pm 1\% \frac{\delta m_{\rm t}}{30 \text{ GeV}} \,.$$

Here the uncertainty of the top quark mass of presently $\delta m_t = 9$ GeV is expected to reach 5 GeV. Similar constraints can be derived from measurements of the forward-backward asymmetry, neutral current processes, and the running $\alpha_{\rm em}$. Thus, any virtuall effects of new physics on $\sigma(e^+e^- \to V_1V_2)$ has to be compared to the ones on Δr, etc. In this paper we will focus on the minimal supersymmetric extension of the SM (MSSM)[3].

2. RC due to Squarks/Sleptons

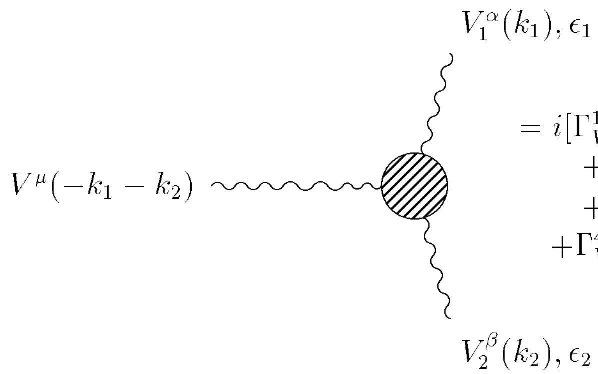
The calculation of the RC to the gauge boson production in e^+e^- collider in the SM is quite elaborate[4)5)]. Thus, in extending this calculation to the MSSM we restrict ourselves to the dominant effects expected to arise from the squark/slepton sector which are enhanced by

- colour factor, $N_c = 3$
- number of generations, $N_g = 3$
- large top Yukawa coupling.

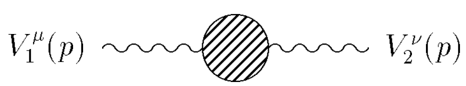
None of these enhancements exist for the selectron-chargino loops. Thus, we will rely throughout this work on the assumption that these contribution can be neglected. (For a more complete treatment see ref. 6).

One consequence of our assumption is that virtuall SUSY contributions only enter via the gauge boson self energy and the TGC. (In this sence, our approach is similar to ref. 7 where the supersymmetric RC to the TGC were calculated). As a result, the amplitude can be written as $\mathcal{A}(\sigma,\epsilon_1,\epsilon_2,s,t) = \sum \mathcal{M}_i^\sigma(\epsilon_1,\epsilon_2) F_i^\sigma(s,t)$ with the matrix elements ($\sigma = \pm 1/2$; $P = 1/2 + \sigma\gamma_5$)[4]

$$\begin{aligned}
\mathcal{M}_1^\sigma(\epsilon_1,\epsilon_2) &= \bar v(p_+) \not\epsilon_+^* (\not k_+ - \not p_+) \not\epsilon_-^* P u(p_-) \\
\mathcal{M}_2^\sigma(\epsilon_1,\epsilon_2) &= \frac{1}{2} \bar v(p_+) (\not k_+ - \not k_-)(\epsilon_+^* \cdot \epsilon_-^*) P u(p_-) \,, \\
\mathcal{M}_3^\sigma(\epsilon_1,\epsilon_2) &= \bar v(p_+) [\not\epsilon_+^* (\epsilon_-^* \cdot k_+) - \not\epsilon_-^* (\epsilon_+^* \cdot k_-)] P u(p_-) \,, \\
\mathcal{M}_4^\sigma(\epsilon_1,\epsilon_2) &= \bar v(p_+) [\not\epsilon_+^* (\epsilon_-^* \cdot p_-) - \not\epsilon_-^* (\epsilon_+^* \cdot p_+)] P u(p_-) \,, \\
\mathcal{M}_5^\sigma(\epsilon_1,\epsilon_2) &= \frac{1}{2} \bar v(p_+) (\not k_+ - \not k_-) P u(p_-) (\epsilon_+^* \cdot k_-)(\epsilon_-^* \cdot k_+) \,, \\
\mathcal{M}_6^\sigma(\epsilon_1,\epsilon_2) &= \bar v(p_+) [\not\epsilon_+^* (\epsilon_-^* \cdot k_+) + \not\epsilon_-^* (\epsilon_+^* \cdot k_-)] P u(p_-) \,.
\end{aligned}$$



$$
\begin{aligned}
= i[&\Gamma^1_{VV_1V_2} k_2^{\alpha} k_1^{\beta} (k_1-k_2)^{\mu} \\
&+\Gamma^2_{VV_1V_2} g^{\alpha\beta} (k_1-k_2)^{\mu} \\
&+\Gamma^3_{VV_1V_2} (g^{\mu\alpha} k_1^{\beta} - g^{\mu\beta} k_2^{\alpha})] \\
&+\Gamma^4_{VV_1V_2} (\epsilon^{\alpha\beta\delta\mu} (k_1-k_2)_{\delta})
\end{aligned}
$$



$$
= i[g^{\mu\nu} A_{V_1V_2}(p^2) + p^{\mu} p^{\nu} B_{V_1V_2}(p^2)]
$$

Fig. 2. The Lorentz invariant decompositions of the gauge boson self-energies and the TGC. The ellips in the first diagram stands for all contributions proportional to k_1^{α}, k_2^{β}, $(k_1+k_2)^{\mu}, \ldots$.

The formfactors for $e^+e^- \to W^+W^-$ are

$$
\begin{aligned}
F_1^{-1/2} &= \frac{g^2}{2t}\left(1+2\frac{\delta g}{g}\right) - 4\frac{g^L_{eeZ}\Gamma^4_{ZW^+W^-}}{s-m_z^2} + 4\frac{e\Gamma^4_{\gamma W^+W^-}}{s}\,, \\
F_1^{+1/2} &= 4\frac{g^R_{eeZ}\Gamma^4_{ZW^+W^-}}{s-m_z^2} - 4\frac{e\Gamma^4_{\gamma W^+W^-}}{s}\,, \\
F_2^{\sigma} &= 2\frac{g^{\sigma}_{eeZ} g_{ZW^+W^-}}{s-m_z^2}\left(1+\frac{\delta g^{\sigma}_{eeZ}}{g^{\sigma}_{eeZ}} + \frac{\delta g_{ZW^+W^-}}{g_{ZW^+W^-}} + \frac{A_{ZZ}(s)-A_{ZZ}(m_z^2)}{s-m_z^2} - \delta Z_{ZZ}\right. \\
&\qquad \left. -\frac{e}{g_{ZW^+W^-}}\frac{A^{ren}_{\gamma Z}(s)}{s} + \frac{\Gamma^2_{ZW^+W^-}+4\sigma\Gamma^4_{ZW^+W^-}}{g_{ZW^+W^-}}\right) \\
&\qquad -2\frac{e^2}{s}\left(1+2\frac{\delta e}{e} + \frac{A_{\gamma\gamma}(s)}{s} - \delta Z_{\gamma\gamma} + \frac{g_{ZW^+W^-}}{e}\frac{A^{ren}_{\gamma Z}(s)}{s-m_Z^2} + \frac{\Gamma^2_{\gamma W^+W^-}+4\sigma\Gamma^4_{\gamma W^+W^-}}{e}\right)\,, \\
F_3^{\sigma} &= -F_2^{\sigma} - \frac{\Gamma^2_{ZW^+W^-}+\Gamma^3_{ZW^+W^-}+2\sigma\Gamma^4_{ZW^+W^-}}{g_{ZW^+W^-}} - \frac{\Gamma^2_{\gamma W^+W^-}+\Gamma^3_{\gamma W^+W^-}+2\sigma\Gamma^4_{\gamma W^+W^-}}{e}\,, \\
F_4^{\sigma} &= -4\sigma\frac{g^{\sigma}_{eeZ}\Gamma^4_{ZW^+W^-}}{s-m_z^2} - 4\sigma\frac{e\Gamma^4_{\gamma W^+W^-}}{s}\,, \\
F_5^{\sigma} &= -2\frac{g^P_{eez}\Gamma^1_{WWZ}}{s-m_z^2} + 2\frac{e\Gamma^1_{WW\gamma}}{s} \qquad P = L, R\,,
\end{aligned} \tag{2}
$$

where $g^{+1/2}_{eeZ} = g\sin^2\theta_{\rm w}/\cos\theta_{\rm w}$, $g^{-1/2}_{eeZ} = g^{+1/2}_{eeZ} + g/2\cos\theta_{\rm w}$, $g^{\sigma}_{ZWW} = g\cos\theta_{\rm w}$. The formfactors for $e^+e^- \to V_1V_2$ $(V_{1/2} = \gamma, Z)$ are . Note that for squark-loops the factors $\Gamma^4_{VW^+W^-}$ and all $\Gamma^i_{VV_1V_2}$ defined in fig. 2 vanish.

3. Numerical Result

Our sfermion mass spectrum is characterized by a universal mass parameter m_0 (here, we omit the possibility of mass-splitting due to renormalization group evolution), a L/R mixing parameter A_0 and the ratio of Higgs VEVs $\tan\beta$. In fig. 3 we see that the differential cross-sections with γ's in the final state diverges for $|\cos\theta| = 1$. In this case, we choose the range

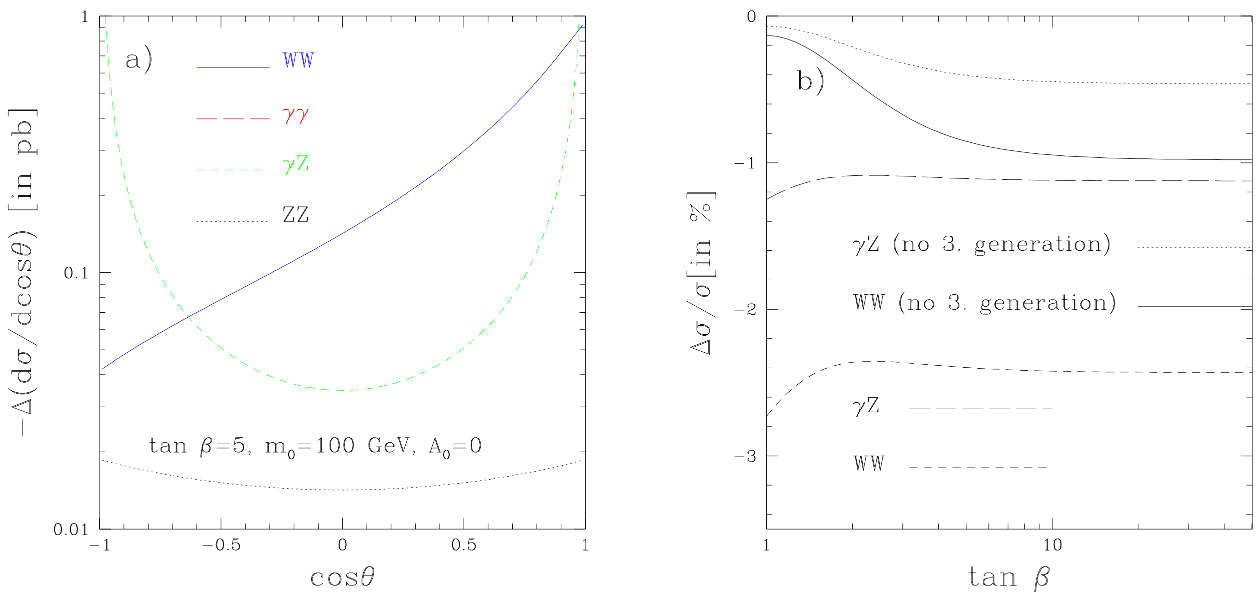


Fig. 3. differential and total cross-sections vs. $\cos\theta$ and $\tan\beta$, respectively. In b) results are presented with and without the 3. generation squarks for $E_{cm} = 190$ GeV.

of integration as $|\cos\theta| < 0.95$. Furthermore, we find that the dominant RC come indeed from the third generation squarks due to large mass splitting within SU(2) multiplet. For the superpartners of the light fermions this splitting is generated by the $\mathrm{SU}(2)_L$ D term, $M^2_{u_L} - M^2_{d_L} = g\langle D^3\rangle = m^2_{\mathrm{w}}\cos 2\beta$.

The counter terms in eq. 2 depend on the renormalization scheme, ie. the parameterization of the Born term. Sofar we have used $g^2 = 4\pi\alpha_{\mathrm{em}}/\sin^2\theta_{\mathrm{w}}$ It is easy to understand that the RC to $e^+e^- \to \gamma\gamma$ vanish in this scheme while those to $e^+e^- \to W^+W^-$ can become quite large (see fig. 4b). However, if we change the renormalized coupling constant to $g^2 \to \sqrt{2}m^2_{\mathrm{w}}G_\mu$ then we have to replace $\delta g/g \to \delta g/g + \Delta r$ and $\Delta\sigma/\sigma \to \Delta\sigma/\sigma - 2\Delta r$. In this scheme the RC to $e^+e^- \to W^+W^-$ are very tiny (fig. 4b). For $\sigma(e^+e^- \to ZZ)$ and $\sigma(e^+e^- \to \gamma Z)$ the situation is similar. In the SM it is convenient to use $G_\mu, \alpha_{\mathrm{em}}, m_{\mathrm{z}}$ and possibly m_{w} to parameterize the tree-level term because these observables are know to such a high precision. However, in the MSSM the largest uncertainty arise from our ignorance of the SUSY parameters. Thus, it is convenient to parameterize in such a way that RC cancel, even if that means to use observables with larger errors. Eg.: it is easy to see that no sfermionic RC exist to the relation

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to \gamma Z) = \alpha_{\mathrm{em}}\Gamma(Z \to e^+e^-) \times \ldots,$$

where the ellips stands for some kinematical factors. Thus, with an error of 0.3%[1)] in the leptonic width of the Z boson an expected precision of at best 0.4% for $\sigma(e^+e^- \to \gamma Z)$ at NLC this process is not suited to yield new information on sfermions. A similar relation holds between $\sigma(e^+e^- \to ZZ)$, $\Gamma(Z \to e^+e^-)$, and $\Gamma(Z \to hadrons)$. A deviation of the TGC from the SM prediction (assuming Δr agrees with the SM prediction) will indicate new physics other than the MSSM.

4. Summary

We have investigated virtual effects of sfermions on the cross-sections of gauge boson pair productions. We find that

- the genuine vertex corrections of sfermions to ZW^+W^- and γW^+W^- are very small;

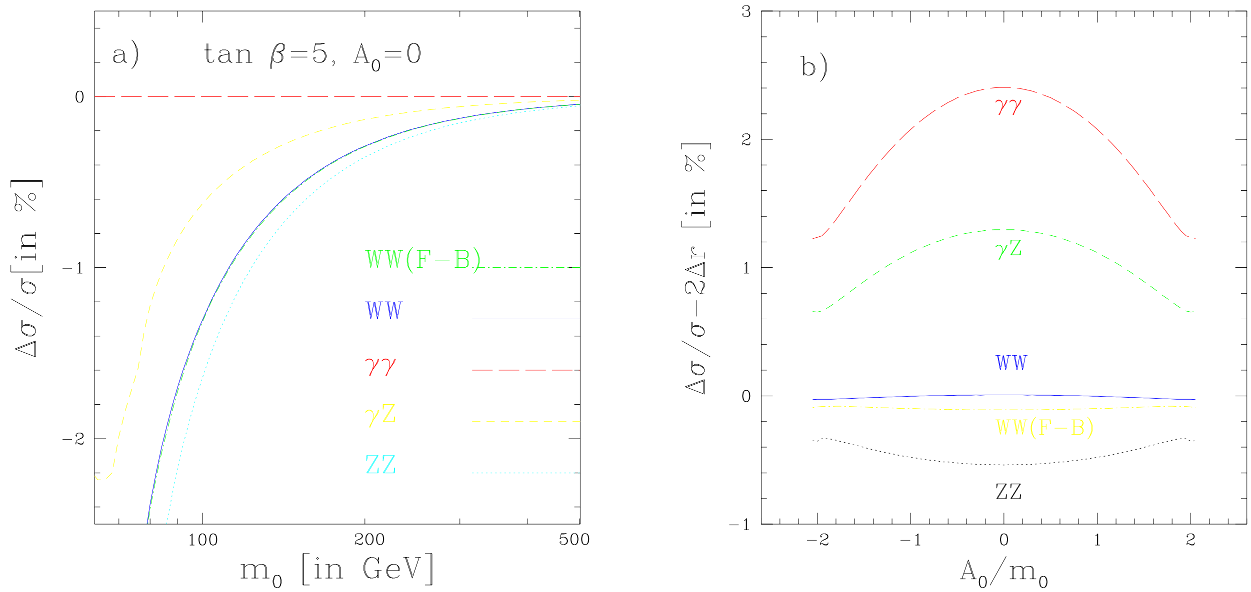


Fig. 4. the sfermionic RC to the total cross-sections vs. m_0 and A_0. The born term is parameterized by a) $\alpha_{\rm em}$ and b) G_μ.

- the genuine vertex corrections of sfermions to $V_1V_2V_3$ vanish if $V_i = \gamma, Z$;
- all oblique corrections can be absorbed by a suitable parametrization of the Born term;
- the experimental precision on $\sigma(e^+e^- \to V_1V_2)$ is lower than the precision of the corresponding EW observables $\Gamma(Z \to hadrons)$ or $\Gamma(Z \to leptons)$ etc;
- sfermions cannot explain any anomalous TGC that may be observed at LEP-II unless there is also evidence of new physics in other observables (e.g. ΔR);
- if a deviation in e.g. Δr is found then $\sigma(e^+e^- \to V_1V_2)$ provides an additional cross-check;

5. Acknowledgements

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